

Dynamical localization of gauge fields on a brane

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Abstract

We propose a dynamical mechanism of localization of gauge fields on a brane in which gauge bosons are excitations of the brane itself or composites made out of matter fields localized on the brane. The mechanism is operative for both Abelian and non-Abelian gauge fields. Several scalar and scalar-fermion composite models of gauge fields are considered. The models exhibit exact gauge invariance and therefore charge universality of gauge interactions is automatically preserved. The mechanism is shown to be equivalent to a modification of the Dvali, Gabadadze and Shifman scenario in which gauge bosons have no bulk kinetic terms and only possess induced kinetic terms on the brane.

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1 Introduction

In the brane world scenarios with large or infinite extra dimensions [1 – 8] one has to explain why we live on the brane and do not escape into extra dimensions. In other words, one needs a mechanism by which the ordinary (standard-model) matter is trapped on the brane while only gravity and possibly some other particles which are the singlets of the standard model can propagate in the bulk.

While simple mechanisms of trapping scalars and fermions have been constructed [1, 2, 3], localizing gauge fields on the brane is notoriously difficult. The main problem turned out to be preserving charge universality of gauge interactions. The interactions of the localized charged particles with gauge fields depend in general not only on their charges but also on their wave functions in the directions transverse to the brane, thus violating charge universality [9, 10]. Several mechanisms of localization of gauge fields on a brane which evade this difficulty have been suggested so far, both in flat [11] and warped [12, 13] space-times. Nongravitational mechanisms are of particular interest as in some popular brane world scenarios extra dimensions are flat [1, 4, 6].

In the present Letter we propose a simple mechanism of localization of gauge fields on a brane which does not rely on gravity and so can work in both flat and warped space-times. It is operative for Abelian as well as non-Abelian gauge fields. In our mechanism gauge fields are composites made out of localized scalar or fermion fields. The localized matter fields can be either fluctuations of the brane itself, in which case the composite gauge fields are the massless vector excitations of the brane, or they can be other zero-mode scalar or fermion fields localized on the brane. We show that in pure fermionic composite models of gauge fields gauge invariance cannot be naturally implemented, while scalar and scalar-fermion models can be made gauge invariant, thus preserving charge universality automatically. We demonstrate how the higher-dimensional gauge invariance translates into the exact gauge invariance of the effective four-dimensional theory irrespective of the shapes of the localization wave functions of the matter fields.

We also show that our mechanism is formally equivalent to a modification of the Dvali, Gabadadze and Shifman (DGS) scenario [14]. In [14] a mechanism of quasi-localization of gauge fields on a brane was proposed, in which gauge fields, in addition to bulk kinetic terms, have induced kinetic terms on the brane. The gauge fields are localized and their interactions are essentially four-dimensional at distances small compared to a crossover scale r_c , while at distances larger than r_c gauge interactions are higher-dimensional and gauge fields can escape to the bulk. It has been argued in [10] that, while this scenario is viable in a five-dimensional space-time, it may have problems when the number of extra dimensions $d \geq 2$: in the case of the δ -function type brane the gauge boson propagator does not exist, while for finite-thickness branes charge universality cannot be preserved. Our mechanism is essentially equivalent to a modification of the DGS scenario in which gauge fields have only induced kinetic terms on the brane and no bulk kinetic terms. The mechanism thus leads to the exact localization of gauge fields on a brane rather than to quasi-localization. In addition,

because of the absence of the gauge boson kinetic terms in the bulk, the propagators of the gauge bosons exist and charge universality is preserved in space-times with an arbitrary number of extra dimensions d and for both δ -function type and finite-thickness branes.

2 Fermionic models

Fermionic composite models of gauge fields have been widely discussed in the literature. Most of them are based on the Bjorken model [16] with the nonlinear Lagrangian

$$\mathcal{L}(\psi, \bar{\psi}) = \bar{\psi}(i\bar{\partial} - M)\psi - G(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi). \quad (1)$$

The standard technique of dealing with such a nonlinear model is to linearize it by introducing an auxiliary vector field A_μ . Indeed, the generating functional in the model

$$Z_1 = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{i\int \mathcal{L}(\psi, \bar{\psi}) d^4x} \quad (2)$$

can be rewritten as

$$Z_2 = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A e^{i\int \mathcal{L}(\psi, \bar{\psi}, A) d^4x} \quad (3)$$

where

$$\mathcal{L}(\psi, \bar{\psi}, A) = \bar{\psi}(i\bar{\partial} - M)\psi - e_0\bar{\psi}\gamma_\mu\psi A^\mu + \frac{m_0^2}{2}A_\mu A^\mu. \quad (4)$$

The path integral over A_μ in (3) is Gaussian, and by performing it one recovers the generating functional Z_1 of eq. (2) with the identification

$$G = \frac{e_0^2}{2m_0^2}. \quad (5)$$

The Lagrangian in eq. (4) describes a spin-1/2 field interacting with the vector field A_μ . The theory is reminiscent of the spinor QED except that the field A_μ has a mass term which breaks gauge invariance, and does not have a kinetic term. The non-propagating classical auxiliary field A_μ acquires the kinetic term through quantum fluctuations of the fermion field and so becomes a physical propagating field [17, 18]; at one fermion loop level one finds

$$\mathcal{L}_{kin} \simeq -\frac{e_0^2}{12\pi^2} \ln(\Lambda^2/M^2) \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \equiv -Z_3 \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (6)$$

where Λ is the ultraviolet cutoff, and after the renormalization $A_\mu \rightarrow \sqrt{Z_3}A_\mu$ one gets the standard spinor QED with a massive photon field.

The nonvanishing photon mass $m_0 \neq 0$ in eq. (4) is clearly related to the fact that the original Lagrangian (1) is not in general gauge invariant. There have been several suggestions of how to deal with this problem. One possibility [19] is to consider the limit $m_0 \rightarrow 0$ which

through eq. (5) is equivalent to $G \rightarrow \infty$. This, however, does not appear to be a satisfactory solution. It is easy to see that the Lagrangian (1) indeed becomes gauge invariant in this limit, but at the expense of neglecting the gauge-noninvariant kinetic term compared to the current-current term which has local $U(1)$ symmetry. This means that the fermionic field ψ becomes an unphysical non-propagating field in this limit; in particular, the gauge boson kinetic term can no longer be generated through the fermion loops. An alternative suggestion [20] was to require that the current $j_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$ vanish identically, which makes the kinetic term in the fermionic Lagrangian gauge invariant. However, in this case one obtains a non-interacting gauge boson field, which is not of much interest. Yet another possibility [21] is to cancel the photon mass term in (4) against the gauge-noninvariant $\mathcal{O}(\Lambda^2)$ contribution coming from the one-fermion-loop self energy of photon calculated with the Euclidean momentum cutoff. In this approach one considers the photon mass term as a counter term introduced to compensate for the use of a gauge-noninvariant regularization. This, however, seems to be rather artificial as gauge invariance does not follow from the form of the Lagrangian but is rather imposed on the theory “by hand”. In addition, the argument does not apply if one employs a gauge-invariant regularization.

To summarize, the fermionic models are not quite satisfactory as they have difficulties ensuring gauge invariance of the induced gauge boson theory. They may, however, be useful if one considers gauge invariance as an approximate symmetry valid only at distances small compared to the scale $R \sim m_0^{-1}$. It is not difficult to construct a higher-dimensional generalization of the Lagrangian (1) with the fermionic chiral zero mode Ψ localized on a 3-dimensional brane. For example, in a five-dimensional space-time one can write

$$\mathcal{L}(\Psi, \bar{\Psi}) = \bar{\Psi}i\Gamma^B\partial_B\Psi + \Delta\mathcal{L} - G_{(5)}(\bar{\Psi}\Gamma^B\Psi)(\bar{\Psi}\Gamma_B\Psi). \quad (7)$$

Here $\Psi(x, z) = u(z)\psi(x)$, x^μ ($\mu = 0, 1, 2, 3$) and z are the coordinates along the brane and in the transverse (fifth) direction respectively, Γ_B ($B = 0, 1, 2, 3, 5$) are the five-dimensional gamma matrices: $\Gamma_\mu = \gamma_\mu$, $\Gamma_5 = -i\gamma_5$, and $\Delta\mathcal{L}$ describes the fermion-brane interaction. The localization wave function $u(z)$ falls off at the distances $|z| \sim m^{-1}$ where m^{-1} is the brane thickness. The model can be linearized by introducing an auxiliary 5-vector field $\mathcal{A}_B = (\mathcal{A}_\mu, \mathcal{A}_5)$. At the one fermion loop level the field \mathcal{A}_B acquires a gauge-invariant kinetic term which is localized on the brane because the fermions are trapped there.

The model sketched above is not in general gauge invariant and therefore may have problems ensuring charge universality of gauge interactions. We therefore will concentrate on scalar and scalar-fermionic models in which exact gauge invariance can be naturally implemented.

3 Scalar and scalar-fermion models in four dimensions

The origin of gauge-noninvariance of the pure fermionic models discussed above can be traced back to the quadratic in A_μ terms in the auxiliary Lagrangians. Such quadratic terms

are necessary for the path integrals over A_μ to be Gaussian, and in fermionic theories they are nothing but the mass terms of the auxiliary vector fields which break gauge invariance. In contrast to this, in scalar theories A_μ^2 terms do not in general break gauge invariance; moreover, such terms are actually necessary to ensure gauge invariance.

We shall consider the nonlinear scalar model with the Lagrangian

$$\mathcal{L}(\phi, \phi^\dagger) = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi^\dagger \phi) - \frac{(i\phi^\dagger \overleftrightarrow{\partial}^\mu \phi)(i\phi^\dagger \overleftrightarrow{\partial}_\mu \phi)}{4\phi^\dagger \phi}. \quad (8)$$

This Lagrangian is invariant with respect to the local $U(1)$ transformation $\phi \rightarrow e^{i\alpha(x)}\phi$ despite the absence of the gauge fields¹. The model can be linearized with the help of the auxiliary vector field A_μ , the Lagrangian of the model being

$$\mathcal{L}(\phi, \phi^\dagger, A) = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi^\dagger \phi) - e_0(i\phi^\dagger \overleftrightarrow{\partial}_\mu \phi)A^\mu + e_0^2 \phi^\dagger \phi A_\mu A^\mu. \quad (9)$$

The last (quadratic in A_μ) term is not an A_μ mass term but rather is the $\phi\phi AA$ coupling which is required by gauge invariance. Integrating over A_μ in the path integral one arrives at the generating functional of the model (8).

The Lagrangian (9) describes scalar QED without the kinetic term of the photon field. At the classical level the equation of motion of A_μ expresses it in terms of the scalar field:

$$A_\mu = \frac{1}{2e_0} \frac{i\phi^\dagger \overleftrightarrow{\partial}_\mu \phi}{\phi^\dagger \phi}. \quad (10)$$

The field (10) has the correct transformation properties under the $U(1)$ gauge transformation, $A_\mu \rightarrow A_\mu - (1/e_0) \partial_\mu \alpha(x)$ (notice that this is not so in the fermionic case). Quantum fluctuations of the scalar field induce the usual gauge-invariant kinetic term for A_μ . At one loop level two diagrams contribute, yielding

$$\mathcal{L}_{kin} \simeq -\frac{e_0^2}{48\pi^2} \ln(\Lambda^2/\mu^2) \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \equiv -Z_3 \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (11)$$

where Λ and μ are the ultraviolet and infrared cutoffs, respectively. After the renormalization $A_\mu \rightarrow \sqrt{Z_3} A_\mu$ one obtains the standard Lagrangian of scalar QED. Notice that the renormalized parameters do not depend on the redundant parameter e_0 , the renormalized charge being

$$e^2(\mu) = \frac{48\pi^2}{\ln(\Lambda^2/\mu^2)}. \quad (12)$$

The fact that there is no charge parameter in the original Lagrangian (8) and the physical charge is generated dynamically is related to the circumstance that the kinetic term of the gauge field is generated dynamically.

¹Scalar theories possessing local gauge invariance and generating gauge fields dynamically have been previously discussed in the framework of the non-linear sigma model (see, e.g., [15]). In contrast to these models, we do not impose any constraints on the scalar field ϕ in (8).

A comment is in order at this point. In eqs. (6) and (11) and in similar formulas below we neglect the terms of order unity as well as terms containing positive powers of p^2/Λ^2 (where p is an external momentum which we assume to be small compared to Λ) and only retain $\log \Lambda$ terms. While the logarithmic terms are universal, $\mathcal{O}(1)$ and smaller terms depend on the details of the regularization scheme used and, with the ultraviolet cutoff Λ in place, even on the momentum routing along the loops. These model-dependent terms can be neglected if $\log \Lambda$ terms are large, which we assume.

We have demonstrated that physical gauge bosons can be generated dynamically in nonlinear scalar models with Lagrangians of the type (8). Several questions then naturally arise:

- Can the model be generalized to the case of several scalar fields with different charges?
- Can charged fermions be incorporated in this scenario?
- Can non-Abelian gauge fields be generated in a similar way?

We shall now answer these questions in turn.

Assume that we have n scalar fields with the charges e_i assembled into a vector $\phi = (\phi_1, \dots, \phi_n)$. The $U(1)$ gauge transformation for ϕ is $\phi \rightarrow e^{iq\alpha(x)}\phi$, where q is the matrix of the charges. It is then easy to see that the Lagrangian

$$\mathcal{L}(\phi, \phi^\dagger) = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi^\dagger \phi) - \frac{(i\phi^\dagger \overleftrightarrow{\partial}^\mu \phi)(i\phi^\dagger q \overleftrightarrow{\partial}_\mu \phi)}{4\phi^\dagger q^2 \phi} \quad (13)$$

has the local $U(1)$ symmetry. The application of the auxiliary field formalism is then straightforward; the model is equivalent to the usual QED of n charged scalar fields.

Once the model contains scalars so that the A_μ^2 terms in the auxiliary Lagrangians are gauge invariant, one can easily incorporate fermions. For example, in the case of one scalar and one spinor field the nonlinear Lagrangian of the model is

$$\mathcal{L}(\phi, \phi^\dagger, \psi, \bar{\psi}) = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi^\dagger \phi) + \bar{\psi}(i\partial - M)\psi - \frac{(i\phi^\dagger \overleftrightarrow{\partial}_\mu \phi + \bar{\psi}\gamma_\mu \psi)^2}{4\phi^\dagger \phi} \quad (14)$$

One can readily make sure that it is gauge invariant. The auxiliary vector field is introduced in the usual way. At the classical level, its equation of motion expresses it through the scalar and spinor fields:

$$A_\mu = \frac{1}{2e_0} \frac{(i\phi^\dagger \overleftrightarrow{\partial}_\mu \phi + \bar{\psi}\gamma_\mu \psi)}{\phi^\dagger \phi}. \quad (15)$$

This field has the correct transformation properties under the $U(1)$ gauge transformation. The field A_μ becomes a physical propagating photon field after its kinetic term is induced by scalar and fermion loops. The resulting theory is the QED with scalar and spinor fields. It is easy to generalize the above model to the case of an arbitrary number of scalar and fermion fields with in general different electric charges.

The mechanism under discussion can be used to generate non-Abelian composite gauge fields as well. Consider the $SU(2)$ case as an example. Let the scalar field ϕ be in the fundamental representation; then the Lagrangian

$$\mathcal{L}(\phi, \phi^\dagger) = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi^\dagger \phi) - \frac{(i\phi^\dagger \overleftrightarrow{\tau} \partial^\mu \phi)(i\phi^\dagger \overleftrightarrow{\tau} \partial_\mu \phi)}{4\phi^\dagger \phi} \quad (16)$$

possesses the local $SU(2)$ symmetry. It can be linearized with the auxiliary vector field A_μ^i in the adjoint representation. Since vector-scalar interactions are gauge invariant, the full gauge invariant kinetic term for A_μ^i is induced through one-scalar-loop diagrams; triple and quartic couplings come from the three-point and four-point A_μ^i functions, respectively. These functions have the same logarithmic renormalization factor $\tilde{Z} = Z_3$. One also obtains higher-dimension terms through one-loop diagrams with more than four external A_μ^i legs; however, it is easy to see that all these terms are either of order unity or contain positive powers of p^2/Λ^2 and so we neglect them.

4 Higher-dimensional models

We shall now consider higher-dimensional composite gauge boson models and discuss the localization of the gauge fields on three-dimensional branes. As our mechanism does not rely on gravity, we consider flat space-times. For definiteness, we consider models in $4 + 1$ dimensions; the generalization to the case of $d > 1$ extra dimensions is straightforward.

We start with the case of a single scalar zero mode Φ localized on a brane. The field Φ can be either a small fluctuation of the brane itself [1], or an independent localized scalar field. The Lagrangian of the model is

$$\mathcal{L}_{(5)}(\Phi, \Phi^\dagger) = \partial^B \Phi^\dagger \partial_B \Phi + \Delta \mathcal{L} - \frac{(i\Phi^\dagger \overleftrightarrow{\partial}^B \Phi)(i\Phi^\dagger \overleftrightarrow{\partial}_B \Phi)}{4\Phi^\dagger \Phi}. \quad (17)$$

Here $\Phi(x, z) = \varphi(z)\phi(x)$. The (real) localization wave function $\varphi(z)$ falls off at the distances $|z| \sim m^{-1}$ from the brane, m^{-1} being the brane thickness. It is normalized by the condition

$$\int_{-\infty}^{\infty} dz \varphi^2(z) = 1. \quad (18)$$

The term $\Delta \mathcal{L}$ describes the interaction of the zero mode Φ with the brane; it cancels the term $\sim m^2 \varphi^2(z) \phi^\dagger \phi$ coming from the derivative over z in the kinetic term: ²

$$\partial^B \Phi^\dagger \partial_B \Phi + \Delta \mathcal{L} = \varphi^2(z) \partial^\mu \phi(x)^\dagger \partial_\mu \phi(x). \quad (19)$$

²For example, if the brane is described by a kink $\Phi_0(z) = (m/\sqrt{\lambda}) \tanh(mz/\sqrt{2})$ [1], the linearized equation of motion for small fluctuations $\Phi(x, z)$ of the brane, $[\partial^B \partial_B - m^2 + 3\lambda \Phi_0(z)^2] \Phi = 0$, has a localized zero-mode solution $\Phi(x, z) = \varphi(z)\phi(x)$ with the normalized localization wave function $\varphi(z) = (3m)^{1/2}/2^{5/4} \cosh^{-2}(mz/\sqrt{2})$. From the equation of motion for $\Phi(x, z)$ one reconstructs $\Delta \mathcal{L} = (-m^2 + 3\lambda \Phi_0(z)^2) \Phi^\dagger \Phi$, which leads to (19). Eq. (19) is, however, quite general and does not depend on the explicit form of the brane; it just reflects the fact that the localized field is a zero mode.

Since the localization wave function $\varphi(z)$ is real, one has

$$i\Phi^\dagger \overleftrightarrow{\partial}_B \Phi = \varphi^2(z) i\phi(x)^\dagger \overleftrightarrow{\partial}_\mu \phi(x) \delta_{B\mu}. \quad (20)$$

Putting eqs. (17), (19) and (20) together we arrive at

$$\mathcal{L}_{(5)} = \varphi^2(z) \left\{ \partial^\mu \phi^\dagger \partial_\mu \phi - \frac{(i\phi^\dagger \overleftrightarrow{\partial}^\mu \phi)(i\phi^\dagger \overleftrightarrow{\partial}_\mu \phi)}{4\phi^\dagger \phi} \right\}. \quad (21)$$

The effective four dimensional Lagrangian

$$\mathcal{L}_{(4)} = \int_{-\infty}^{\infty} dz \mathcal{L}_{(5)} \quad (22)$$

then coincides with the Lagrangian (8) with $V(\phi^\dagger \phi) = 0$. One can now apply the auxiliary field formalism as discussed in detail in sec. 3 and show that the massless gauge boson field is produced, whose kinetic term is generated by scalar loops. Alternatively, one could apply the auxiliary field formalism already in the five-dimensional theory. The classical five-dimensional auxiliary field

$$\mathcal{A}_B = \frac{1}{2e_0} \frac{i\Phi^\dagger \overleftrightarrow{\partial}_B \Phi}{\Phi^\dagger \Phi} = \frac{1}{2e_0} \frac{i\phi^\dagger \overleftrightarrow{\partial}_\mu \phi}{\phi^\dagger \phi} \delta_{B\mu} = A_\mu \delta_{B\mu} \quad (23)$$

does not depend on the transverse coordinate z and so is not localized on the brane³. At the same time, its loop-induced kinetic term is localized:

$$\mathcal{L}_{kin(5)} = -\varphi^2(z) Z_3 \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \quad (24)$$

This means that the gauge boson field can only propagate on the brane. Thus, our model has a gauge boson field which lives in the bulk but has only an induced kinetic term on the brane. As was pointed out before, such a model is equivalent to a modification of the DGS scenario [14]. The four-dimensional theory obtained after the integration over the fifth coordinate is identical to the theory resulting from the application of the auxiliary field formalism directly in the four-dimensional space-time.

As can be seen from eq. (21), in the case of one scalar field the z -dependence of $\mathcal{L}_{(5)}$ factorizes out. This, however, is not so if there are more than one scalar and/or fermion fields with different localization wave functions. This raises a question of how gauge invariance is preserved in the effective four-dimensional theory. Indeed, for gauge invariance to hold, the coefficients of different terms in $\mathcal{L}_{(4)}$ must have certain fixed relative values, while with

³This, in particular, means that the field (23) is not normalizable. This point, however, should be of no concern as the field (23) is a non-propagating auxiliary field which simply gives an alternative description of the scalar self-coupling. The integration of the Lagrangian $\mathcal{L}_{(5)}$ of eq. (21) over z does not lead to any divergence.

arbitrary localization wave functions one can expect that upon the integration of $\mathcal{L}_{(5)}$ over z these coefficients will take arbitrary values. We shall now show that in fact this is not the case and demonstrate how the gauge invariance is actually preserved in the four-dimensional theory.

Consider first an example of one scalar and one spinor field with the localization wave functions $\varphi(z)$ and $u(z)$ respectively: $\Phi(x, z) = \varphi(z)\phi(x)$, $\Psi(x, z) = u(z)\psi(x)$. We assume $\varphi(z)$ and $u(z)$ to be normalized according to (18). The Lagrangian of the model is

$$\mathcal{L}_{(5)} = \partial^B \Phi^\dagger \partial_B \Phi + \bar{\Psi} i \Gamma^B \partial_B \Psi + \Delta \mathcal{L} - \frac{(i \Phi^\dagger \overleftrightarrow{\partial}_B \Phi + \bar{\Psi} \Gamma_B \Psi)^2}{4 \Phi^\dagger \Phi}. \quad (25)$$

Here $\Delta \mathcal{L}$ is chosen in such a way that

$$\partial^B \Phi^\dagger \partial_B \Phi + \bar{\Psi} i \Gamma^B \partial_B \Psi + \Delta \mathcal{L} = \varphi^2(z) \partial^\mu \phi^\dagger \partial_\mu \phi + u^2(z) \bar{\psi} i \gamma^\mu \partial_\mu \psi.$$

Thus one can rewrite eq. (25) as

$$\mathcal{L}_{(5)} = \varphi^2(z) \partial^\mu \phi^\dagger \partial_\mu \phi + u^2(z) \bar{\psi} i \gamma^\mu \partial_\mu \psi - \frac{[\varphi^2(z) i \phi^\dagger \overleftrightarrow{\partial}_\mu \phi + u^2(z) \bar{\psi} \gamma_\mu \psi]^2}{4 \varphi^2(z) \phi^\dagger \phi}, \quad (26)$$

where we have used the fact that the localized fermionic zero modes are chiral, so that $\bar{\Psi} \Gamma_5 \Psi = \pm \bar{\Psi} \Psi = 0$. The last term in this expression is

$$-\frac{1}{4} \left[\varphi^2(z) \frac{(i \phi^\dagger \overleftrightarrow{\partial}_\mu \phi)^2}{\phi^\dagger \phi} + 2 u^2(z) \frac{(i \phi^\dagger \overleftrightarrow{\partial}_\mu \phi) \bar{\psi} \gamma^\mu \psi}{\phi^\dagger \phi} + \frac{u^4(z) (\bar{\psi} \gamma_\mu \psi)^2}{\varphi^2(z) \phi^\dagger \phi} \right]. \quad (27)$$

The integration of the first two terms in (27) over z yields the correct coefficients for these terms to produce, together with the (integrated) kinetic terms, a gauge invariant expression; the integral of the last term is gauge invariant by itself. Thus we obtain

$$\mathcal{L}_{(4)} = \partial^\mu \phi^\dagger \partial_\mu \phi + \bar{\psi} i \not{\partial} \psi - \frac{(i \phi^\dagger \overleftrightarrow{\partial}_\mu \phi + \bar{\psi} \gamma_\mu \psi)^2}{4 \phi^\dagger \phi} + C \frac{(\bar{\psi} \gamma_\mu \psi)^2}{\phi^\dagger \phi} \quad (28)$$

where

$$C = -\frac{1}{4} \int_{-\infty}^{\infty} dz \frac{u^4(z) - \varphi^4(z)}{\varphi^2(z)}. \quad (29)$$

Except for the last term, the Lagrangian in eq. (28) coincides with that in eq. (14) with $V(\phi^\dagger \phi) = M = 0$. The last term in (28) is a nonlinear gauge-invariant expression. Note that for $\varphi(z) = u(z)$ the constant C vanishes; therefore when the localization wave functions of the spinor and scalar fields coincide, the theory is fully linearized by the dynamical generation of the gauge boson. Otherwise the four-dimensional theory has a residual nonlinear coupling, even though the five-dimensional theory is fully linearized.

Consider now a slightly more complicated case of two localized scalar fields with different localization wave functions, $\varphi_1(z)$ and $\varphi_2(z)$, both normalized according to (18). The five-dimensional Lagrangian of the model is

$$\mathcal{L}_{(5)} = \sum_{i=1,2} \partial^B \Phi_i^\dagger \partial_B \Phi_i + \Delta\mathcal{L} - \frac{\left(i \sum_{i=1,2} \Phi_i^\dagger \overset{\leftrightarrow}{\partial}_B \Phi_i \right)^2}{4 \sum_{j=1,2} \Phi_j^\dagger \Phi_j}, \quad (30)$$

where $\Delta\mathcal{L}$ has been chosen in the usual way. In calculating the integral of $\mathcal{L}_{(5)}$ over z one encounters three types of integrals,

$$\begin{aligned} I_1 &= \int_{-\infty}^{\infty} dz \frac{\varphi_1^4(z)}{A\varphi_1^2(z) + B\varphi_2^2(z)}, \quad I_2 = \int_{-\infty}^{\infty} dz \frac{\varphi_2^4(z)}{A\varphi_1^2(z) + B\varphi_2^2(z)}, \\ I_3 &= \int_{-\infty}^{\infty} dz \frac{\varphi_1^2(z)\varphi_2^2(z)}{A\varphi_1^2(z) + B\varphi_2^2(z)}, \end{aligned} \quad (31)$$

where

$$A \equiv \phi_1(x)^\dagger \phi_1(x), \quad B \equiv \phi_2(x)^\dagger \phi_2(x). \quad (32)$$

Out of these three integrals, only one is independent. For example, one can express I_1 and I_2 through I_3 :

$$I_1 = \frac{1 - BI_3}{A}, \quad I_2 = \frac{1 - AI_3}{B}. \quad (33)$$

Using these relations it is straightforward to check that the corresponding four-dimensional theory is gauge invariant.

5 Discussion and conclusion

We have proposed a simple mechanism of localization of gauge fields on a brane which can work in space-times with an arbitrary number of extra dimensions, both flat and warped. The gauge fields are assumed to be composites made out of zero-mode matter fields localized on the brane. The localized matter fields may acquire masses through a mechanism different from the localization one; this would not destroy gauge invariance of the resulting vector field theory.

We have considered several simple scalar and scalar-fermion models in which gauge bosons are dynamically generated, their kinetic terms being produced by quantum fluctuations of the localized matter fields. The mechanism is operative in both Abelian and non-Abelian cases. While pure fermionic models have difficulties ensuring gauge invariance, in models with scalars exact gauge invariance can be naturally implemented. We demonstrated that the higher-dimensional gauge invariance translates into the exact gauge invariance of the effective four-dimensional theory irrespective of the details of the localization mechanism

of matter fields. Charge universality of gauge interactions is thus automatically preserved in the four-dimensional theory. One can expect that a similar mechanism can also localize gravity on a brane.

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